

# Kinetic instability of ion acoustic mode in permeating plasmas

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**Abstract:** In plasmas with electron drift (current) relative to static ions, the ion acoustic wave is subject to the kinetic instability which takes place if the directed electron speed exceeds the ion acoustic speed. The instability threshold becomes different in the case of one quasi-neutral electron-ion plasma propagating through another static quasi-neutral (target) plasma. The threshold velocity of the propagating plasma may be well below the ion acoustic speed of the static plasma. Such a current-less instability may frequently be expected in space and astrophysical plasmas.

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In an un-magnetized plasma with a macroscopic velocity of the electron component  $\vec{v}_{e0} = v_{e0}\vec{e}_z$  relative to static singly-charged ions, a kinetic instability of the ion acoustic mode sets in, described by the growth rate<sup>1</sup>

$$\omega_i = \left(\frac{\pi}{8}\right)^{1/2} k c_s \left[ \left(\frac{m_e}{m_i}\right)^{1/2} \left(\frac{v_{e0}}{c_s} - 1\right) - \tau^{3/2} \exp\left(-\frac{\tau}{2}\right) \right] \quad (1)$$

Here,  $c_s = (\kappa T_e/m_i)^{1/2}$  is the ion sound speed, and  $\tau = T_e/T_i$ . The necessary condition

$$v_{e0} > c_s [1 + (m_i/m_e)^{1/2} \tau^{3/2} \exp(-\tau/2)] \quad (2)$$

may imply a rather high electron current magnitude. A similar description of the ion acoustic mode and the growth rate may be obtained also in the case of a magnetized plasma<sup>2</sup> with the equilibrium magnetic field  $\vec{B}_0 = b_0\vec{e}_z$  and for perturbations propagating in the direction of the electron current  $\vec{v}_{e0} = v_{e0}\vec{e}_z$  that is parallel to the magnetic field vector. The instability described by (1) is a purely kinetic, collision-less, electron inertia effect. Its two-fluid counterpart<sup>3,4</sup> however is a strictly electron-collision effect.

The instability in (1) appears due to the Doppler shift term (the one containing  $v_{e0}$ ), which may determine the sign of the growth rate. However, a much lower instability threshold may be obtained in the case of two interpenetrating (permeating) plasmas. This is then a *current-less* instability, where now the Doppler shift of ions appears to play the main role. This will be demonstrated in the forthcoming text. Such permeating plasmas can be created in lab conditions, while in space this happens to be a rather frequent situation, for example in the case of colliding astrophysical clouds, in the propagation of plasmas originating from the explosions of novae and supernovae and moving through the surrounding plasmas, and also in the case of solar and stellar winds. In the case of the solar wind, one electron-ion quasi-neutral component is generated mainly in the polar regions (fast wind), while the other electron-ion quasi-neutral component (slow wind) originates from lower latitudes, yet at large distances from the origin there may be overlapping between the two. A most obvious example of such permeating plasmas is seen also in the solar atmosphere where plasma streams from lower layers are seen continuously propagating along magnetic field structures towards and through the solar corona. From the text and results that follow it will become clear that the present general study of two interpenetrating plasmas comprises also, as special cases, various examples of plasmas containing some additional (electron or ion) species.

In all of these examples we have one fast flowing plasma whose parameters we shall denote by the subscript  $f$ , and one slow moving (target) plasma with the parameters denoted by the subscript  $s$ . In fact, the target plasma can be non-moving (or 'static',

hence the same index 's'), or if it does move with some constant velocity  $\vec{v}_{s0} = v_{s0}\vec{e}_z$ , the corresponding equations for the whole combined plasma system can conveniently be written in the coordinate frame moving with this same velocity. Therefore, in the forthcoming text the corresponding velocity of the other component  $\vec{v}_{f0} = v_{f0}\vec{e}_z$  will be used to describe just the difference between the two.

In the case of singly-charged ions, the quasi-neutrality conditions for the two plasmas in the equilibrium read:

$$n_{fi0} = n_{fe0} = n_{f0}, \quad n_{si0} = n_{se0} = n_{s0}. \quad (3)$$

In general  $n_{s0} \neq n_{f0}$ , and the same holds for the temperatures of the two separate quasi-neutral plasmas  $T_{fe} \neq T_{se}$ ,  $T_{fi} \neq T_{si}$ .

We use the plasma distribution function for the species  $j$ ,

$$f_{j0} = \frac{n_{j0}}{(2\pi)^{3/2}v_{Tj}^3} \exp \left\{ -\frac{1}{2v_{Tj}^2} [v_x^2 + v_y^2 + (v_z - v_{j0})^2] \right\}, \quad (4)$$

where  $n_{j0} = \text{const}$ , and  $v_{Tj}^2 = \kappa T_j/m_j$ . From the linearized Boltzmann kinetic equation for the perturbed distribution function

$$\frac{\partial f_{j1}}{\partial t} + \vec{v} \frac{\partial f_{j1}}{\partial \vec{r}} + \frac{q_j}{m_j} \left( \vec{E}_1 \frac{\partial f_{j0}}{\partial \vec{v}} + \vec{v} \times \vec{B}_0 \frac{\partial f_{j1}}{\partial \vec{v}} \right) = 0,$$

and for longitudinal electrostatic perturbations  $\sim \exp(-i\omega + ikz)$  we obtain

$$f_{j1} = -\frac{iq_j}{m_j} \frac{\vec{E}_1}{\omega - \vec{k}\vec{v}} \frac{\partial f_{j0}}{\partial \vec{v}}.$$

Further, using the Ampère law

$$0 = \mu_0 \vec{j} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t},$$

and the expression for the macroscopic current

$$\vec{j} = \sum_j q_j \int \vec{v} f_j d^3\vec{v}$$

we arrive at

$$1 = \sum_j \frac{\omega_{pj}^2}{n_{j0}\omega v_{Tj}^2} \int_{-\infty}^{\infty} \frac{v_z(v_z - v_{j0})}{\omega - kv_z} f_{j0} dv_x dv_y dv_z. \quad (5)$$

The remaining integrations in (5) are straightforward and the final result is

$$1 + \sum_j \frac{1}{k^2 \lambda_{dj}^2} [1 - Z(b_j)] = 0. \quad (6)$$

Here,  $b_j = (\omega - kv_{j0})/(kv_{Tj})$ ,  $\lambda_{dj} = v_{Tj}/\omega_{pj}$ ,  $\omega_{pj}^2 = q_j^2 n_{j0}/(\varepsilon_0 m_j)$ , and

$$Z(b_j) = \frac{b_j}{(2\pi)^{1/2}} \int_c \frac{\exp(-\zeta^2/2)}{b_j - \zeta} d\zeta$$

is the plasma dispersion function, with the integration over the Landau contour  $c$ . The dispersion equation (6) describes the plasma (Langmuir) and the ion acoustic oscillations.

In the case  $v_{sj0} = 0$ ,  $v_{fj0} = v_{f0}$ , expanding Eq. (6) in the ion acoustic frequency range on conditions

$$kv_{Tsi} \ll |\omega| \ll kv_{Tse}, \quad |\omega - kv_{f0}| \ll kv_{Tfe}, \quad kv_{Tfi}, \quad (7)$$

where  $v_{Tsj}^2 = \kappa T_{sj}/m_j$ ,  $v_{Tfj}^2 = \kappa T_{fj}/m_j$ , we obtain the dispersion equation for the ion acoustic wave

$$\Delta(k, \omega) \equiv 1 + \frac{1}{k^2 \lambda_d^2} - \frac{\omega_{psi}^2}{\omega^2} - \frac{3k^2 v_{Tsi}^2 \omega_{psi}^2}{\omega^4} + i \left( \frac{\pi}{2} \right)^{1/2} \left[ \underbrace{\frac{\omega \omega_{pse}^2}{k^3 v_{Tse}^3}}_c + (\omega - kv_{f0}) \left( \underbrace{\frac{\omega_{pfe}^2}{k^3 v_{Tfe}^3}}_a + \underbrace{\frac{\omega_{pfi}^2}{k^3 v_{Tfi}^3}}_b \right) + \frac{\omega \omega_{psi}^2}{k^3 v_{Tsi}^3} \exp \left( -\frac{\omega^2}{2k^2 v_{Tsi}^2} \right) \right] = 0. \quad (8)$$

Here,  $1/\lambda_d^2 = 1/\lambda_{dse}^2 + 1/\lambda_{dfe}^2 + 1/\lambda_{dfi}^2$ , and  $\lambda_{dse} = v_{Tse}/\omega_{pse}$  etc. In view of the conditions (7) the inertia of the ion acoustic (IA) mode is provided mainly by the ions of the static (target) plasma.

In standard electron-ion plasma with streaming electrons and for  $T_i \ll T_e$  it yields

$$1 + \frac{1}{\lambda_{de}^2 k^2} - \frac{\omega_{pi}^2}{\omega^2} + i(\pi/2)^{1/2} \left\{ \frac{\omega_{pe}^2 (\omega - kv_{e0})}{k^3 v_{Te}^3} + \frac{\omega_{pi}^2 \omega}{k^3 v_{Ti}^3} \exp \left[ -\omega^2 / (2k^2 v_{Ti}^2) \right] \right\} = 0. \quad (9)$$

From this one can further obtain the growth rate (1) and the frequency  $\omega^2 \simeq k^2 c_s^2$ .

The only terms in (8) that may produce a growing mode are those with the Doppler-shift, the two other terms in the imaginary part always give the Landau damping. It is easily seen that the electron part in the Doppler-shifted term is usually negligible because  $a/b = (T_{fi}/T_{fe})^{3/2} (m_e/m_i)^{1/2} \ll 1$ . Assuming also that

$$c/[b(\omega - kv_{f0})] = |\omega/(\omega - kv_{f0})| (n_{s0}/n_{f0}) (T_{fi}/T_{se})^{3/2} (m_e/m_i)^{1/2} \ll 1,$$

the dispersion equation becomes

$$1 + \frac{1}{\lambda_d^2 k^2} - \frac{\omega_{psi}^2}{\omega^2} - \frac{3k^2 v_{Tsi}^2 \omega_{psi}^2}{\omega^4} + i(\pi/2)^{1/2} \left\{ \frac{\omega_{pfi}^2 (\omega - kv_{f0})}{k^3 v_{Tfi}^3} + \frac{\omega_{psi}^2 \omega}{k^3 v_{Tsi}^3} \exp \left[ -\omega^2 / (2k^2 v_{Tsi}^2) \right] \right\} = 0. \quad (10)$$

The frequency of the ion acoustic wave which follows from this is

$$\omega_r^2 = (k^2 \lambda_d^2 \omega_{psi}^2 / 2) [1 + (1 + 12 \lambda_{si}^2 / \lambda_d^2)^{1/2}]. \quad (11)$$

For an ordinary electron-ion plasma with  $s$ -species only, it takes the shape  $\omega_r^2 = (kc_s^2/2) [1 + (1 + 12 T_{si}/T_{se})^{1/2}]$ .

The growth rate from (10),  $\omega_i \simeq -Im\Delta(k, \omega_r)/[\partial(Re\Delta)/\partial\omega]_{\omega \simeq \omega_r}$ , becomes

$$\omega_i = -\left(\frac{\pi}{8}\right)^{1/2} \frac{\omega_r^3}{k^2 v_{Tfi}^2 (1 + 6k^2 v_{Tsi}^2 / \omega_r^2)} \left\{ \frac{\omega_r - kv_{f0}}{kv_{Tfi}} \frac{n_{f0}}{n_{s0}} + \frac{\omega_r}{kv_{Tsi}} \frac{T_{fi}}{T_{si}} \exp\left[-\omega_r^2 / (2k^2 v_{Tsi}^2)\right] \right\}. \quad (12)$$

In the limit  $T_{fe} \gg T_{fi}$ ,  $T_{se} \gg T_{si}$  and setting  $\omega_r \simeq kc_{se}$ , the growth rate becomes

$$\omega_i = \left(\frac{\pi}{8}\right)^{1/2} kc_{se} \left(\frac{T_{se}}{T_{fi}}\right)^{3/2} \left\{ \left(\frac{v_{f0}}{c_{se}} - 1\right) \frac{n_{f0}}{n_{s0}} - \left(\frac{T_{fi}}{T_{si}}\right)^{3/2} \exp\left[-T_{se}/(2T_{si})\right] \right\}. \quad (13)$$

From this the instability threshold is

$$v_{f0} > c_{se} \left[ 1 + \frac{n_{s0}}{n_{f0}} \left(\frac{T_{fi}}{T_{si}}\right)^{3/2} \exp\left(-\frac{T_{se}}{2T_{si}}\right) \right]. \quad (14)$$

Here,  $c_{se}^2 = \kappa T_{se}/m_i$ . Eq. (14) has a remarkable properties. It is seen that here [as compared to Eq. (2)] the first temperature ratio contains only the two *ion* temperatures, and in addition the electron-ion mass ratio vanishes. In fact, the mass ratio can appear, yet in the present case it would contain the *two ion masses* (that are taken as equal). For the singly charged two ion species, instead of  $n_{s0}/n_{f0}$  in Eq. (14) we would have  $(n_{s0}/n_{f0})(m_{fi}/m_{si})$ . Consequently, in the present case the instability threshold may become considerably lower. This is particularly valid for a flowing plasma being accelerated into a less dense but hotter static target plasma. One example of that kind may be the plasma from the lower solar atmosphere propagating through hot upper layers (e.g., spicules, and flows along magnetic loops in general).

However, in general case the threshold velocity  $v_{f0}$  for the currentless instability may in fact be *below* the sound velocity of the target plasma. As an example, the full Eq. (9) is solved numerically by taking the wavelength  $\lambda = 0.3$  m, and  $T_{se} = 10^5$  K,  $T_{si} = 10^3$  K,  $n_{f0} = 10^{17}$  m<sup>-3</sup>, and  $n_{s0}$  is varied in the range shown in Fig. 1. The critical value of the flowing plasma velocity is shown for several values of the temperature of the flowing plasma. For the velocities above the given lines the IA mode becomes growing. For these parameters, in Eq. (11) the term  $\lambda_{si}^2/\lambda_d^2 = 0.07$  and also  $\lambda_d \omega_{psi} = 0.38c_{se}$ . Therefore, contrary to (1) or (14) the instability threshold (c.f. Fig. 1) is in fact well *below* the IA speed of the static (target) plasma. For lower  $T_{fi}$  the instability threshold is reduced because of a lower Landau damping by the flowing ions.

Note that for the used parameters,  $a/b \ll 1$  but  $c/b$  is on the order of unity or larger. Yet, both the mode frequency and the growth rate are well analytically described by Eqs. (11), (12), respectively.

Instead of the conditions (7), the expansion in Eq. (6) could be done using the following limits:

$$kv_{Tsi} \ll |\omega| \ll kv_{Tse}, \quad kv_{Tfi} \ll |\omega - kv_{f0}| \ll kv_{Tfe}. \quad (15)$$

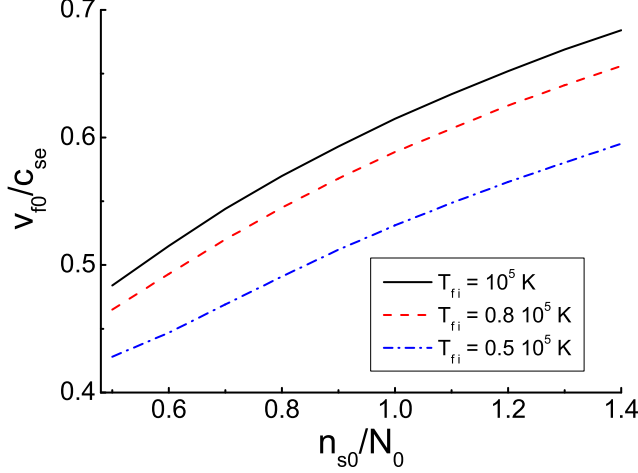


Figure 1: The critical (threshold) values of the flowing plasma velocity for the ion-acoustic wave instability in terms of the number density of the static (target) plasma. Here  $c_{se}^2 = \kappa T_{se}/m_i$ ,  $N_0 = 10^{17} \text{ m}^{-3}$ .

The dispersion equation in this case becomes:

$$\Delta(\omega, k) = 1 + \frac{1}{k^2 \lambda_{dse}^2} + \frac{1}{k^2 \lambda_{dfe}^2} - \frac{1}{k^2 \lambda_{dsi}^2} \frac{1}{b_{si}^2} \left(1 + \frac{3}{b_{si}^2}\right) - \frac{1}{k^2 \lambda_{dfi}^2} \frac{1}{b_{fi}^2} \left(1 + \frac{3}{b_{fi}^2}\right) + i \left(\frac{\pi}{2}\right)^2 \left(\frac{b_{se}}{k^2 \lambda_{dse}^2} + \frac{b_{fe}}{k^2 \lambda_{dfe}^2}\right) + i \left(\frac{\pi}{2}\right)^2 \left[\frac{b_{si}}{k^2 \lambda_{dsi}^2} \exp(-b_{si}^2/2) + \frac{b_{fi}}{k^2 \lambda_{dfi}^2} \exp(-b_{fi}^2/2)\right] = 0. \quad (16)$$

Here,  $b_{fj} = (\omega - kv_{f0})/kv_{Tfj}$ ,  $b_{sj} = \omega/kv_{Tsj}$ . It is seen that as compared to Eq. (10), in the present case the mode is determined by both ion species, while the growth rate is an interplay between the fast ions and fast electrons. The growth rate can easily be discussed in the limit  $T_{fi} = T_{si}$ ,  $T_{se} = T_{fe}$  and for both plasmas with the same density  $n_0$ . In this case the imaginary part of (16) becomes

$$\text{Im}\Delta = \left(\frac{\pi}{2}\right)^2 \left\{ \frac{\omega_{pe}^2}{k^3 v_{Te}^3} + \frac{\omega_{pi}^2}{k^3 v_{Ti}^3} \exp\left[-\omega^2/(2k^2 v_{Ti}^2)\right] \right\} (2\omega - kv_{f0}).$$

If in addition  $\tau \equiv T_e/T_i \gg 1$ , the ratio of the electron and ion terms give the function  $f(\tau) = (m_e/m_i)^{1/2} \tau^{-3/2} \exp(\tau/2)$ . For an electron-proton plasma we have  $f(\tau) > 1$  if  $\tau \geq 16$ , and only in this limit the growth rate is determined by the electron flow term. Otherwise, the IA instability is driven by the ion Doppler shift.

The cases of dominant ion (or electron) terms discussed above are equivalent to studying also electron-ion plasmas containing an additional ion (or electron species).<sup>5-8</sup> As special cases, these include also pair-ion,<sup>9-12</sup> negative-ion,<sup>13</sup> electron-positron-ion plasmas,<sup>14,15</sup> and also dusty plasmas without<sup>16</sup> and with oppositely charged grains.<sup>17</sup> Such plasmas have been extensively studied in the recent past. The general streaming kinetic instability presented above is directly applicable to all these plasmas as special cases.

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